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Recent progress on the rare decay $K_L \to \pi^0 \mu^+ \mu^-$

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Abstract

The rare decay $K_L \to \pi^0 \mu^+ \mu^-$ has a significant CP-conserving contribution. The reliable theoretical estimation of this piece from the experimental $K_L \to \pi^0 \gamma\gamma$ branching ratio is shortly reviewed. Then, we discuss the exceptional sensitivity of the combined set of decays, into $\pi^0 \nu\bar{\nu}$, $\pi^0 e^+ e^-$ and $\pi^0 \mu^+ \mu^-$, to New Physics signal, and also, interestingly, to New Physics nature.

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1 Introduction

Studies of direct CP-violation are important to test the Standard Model, and possibly to discover New Physics effects. Here, we consider the following rare K_L modes

	DCPV	ICPV	CPC
$K_L \to \pi^0 \nu\bar{\nu}$	$\sim 100\%$	$\sim 0\%$	$\sim 0\%$
$K_L \to \pi^0 e^+ e^-$	$\sim 40\%$	$\sim 60\%$	$\sim 0\%$
$K_L \to \pi^0 \mu^+ \mu^-$	$\sim 30\%$	$\sim 35\%$	$\sim 35\%$

The direct CP-violating (DCPV), indirect CP-violating (ICPV) and CP-conserving (CPC) contributions originate from the processes depicted on Fig.1. While the theoretical complexity increases for $\ell^+\ell^-$ modes, recent experimental results for $K_S \to \pi^0 \ell^+ \ell^-$[1] and $K_L \to \pi^0 \gamma\gamma$[2, 3] now permit reliable theoretical estimates for ICPV and CPC, making them competitive with the $\nu\bar{\nu}$ one.

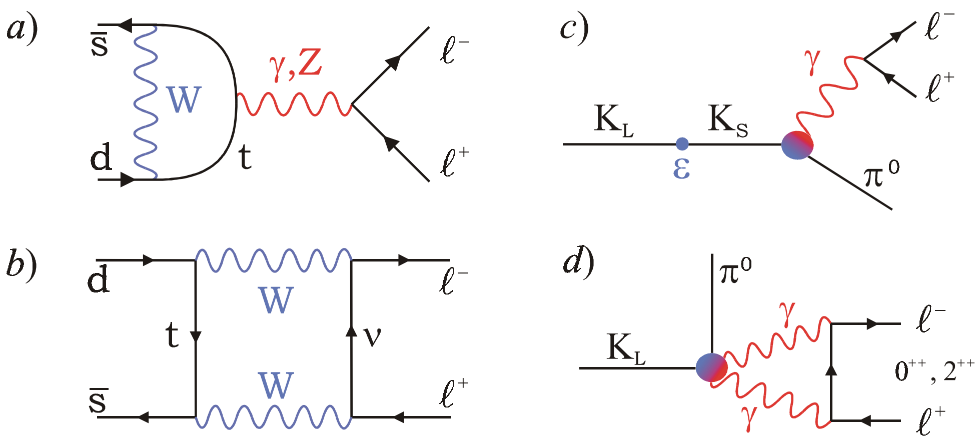

Figure 1: DCPV (a,b), ICPV (c) and CPC (d) contributions.

The CPC contribution, as shown on Fig.1d, proceeds through two photons, which can be in a scalar 0^{++} or tensor 2^{++} state. The former is helicity suppressed, hence contribute only for $\mu^+\mu^-$, while the later, much smaller, is the dominant one for e^+e^- (see Ref.[4]). Our work was to estimate the 0^{++} CPC contribution[5].

2 CP–conserving contribution

At leading order in Chiral Perturbation Theory (i.e. $\mathcal{O}\left(p^4\right)$ here), the process proceeds through a charged meson loop followed by a photon loop, $K_L \to \pi^0 P^+ P^-$, $P^+P^- \to \gamma\gamma \to \ell^+\ell^-$ with $P = \pi, K$, see Fig.2a. In addition, it is factorized and $(P^+P^-)_{0^{++}} \to \gamma\gamma \to \ell^+\ell^-$ can be computed separately. This subprocess is then strictly similar to $K_S \to (\pi^+\pi^-)_{0^{++}} \to \gamma\gamma \to \ell^+\ell^-$ [5, 6].

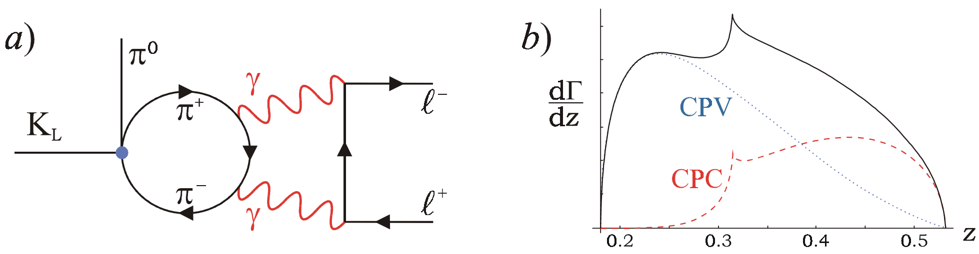

Figure 2: (a) Typical CPC pion loop contribution and (b) the CPV and CPC differential rates for positive interference between DCPV and ICPV.

This factorization holds when one can parametrize the vertex $\mathcal{M}\left(K_L \to \pi^0 P^+ P^-\right) = G_8 m_K^2 a^P(z)$ (see [5, 7]), with $z = \left(p_{P+} + p_{P-}\right)^2 / m_K^2$ the P^+P^- invariant mass and $a^P(z)$ some function ($a^\pi(z) = z - r_\pi^2$ and $a^K(z) = z - r_\pi^2 - 1$ at $\mathcal{O}\left(p^4\right)$). The resulting total rate, taking the $\pi^+\pi^-$ contribution alone for simplicity, can be written as

$$\Gamma_{\ell^+\ell^-} = \frac{G_8^2 m_K^5 \alpha^4}{512\pi^7} \int_{4r_\ell^2}^{(1-r_\pi)^2} dz\ \left|a(z)\right|^2 \lambda_\pi^{1/2} \left|\mathcal{J}\left(\frac{r_\pi^2}{z}, \frac{r_\ell^2}{z}\right)\right|^2 \frac{r_\ell^2}{z}\left(1 - \frac{4r_\ell^2}{z}\right)^{3/2}$$

$$\Gamma_{\gamma\gamma} = \frac{G_8^2 m_K^5 \alpha^2}{4096\pi^5} \int_0^{(1-r_\pi)^2} dz\ \left|a(z)\right|^2 \lambda_\pi^{1/2} \left|\mathcal{J}_{\gamma\gamma}\left(\frac{r_\pi^2}{z}\right)\right|^2$$

where $r_x = m_x/m_K$, $\lambda_\pi = \lambda\left(1, r_\pi^2, z\right)$ is the standard two-body kinematical function for $\pi^0 + (\pi^+\pi^-)_{0^{++}}$ in a $L = 0$ wave, r_ℓ^2/z is the helicity suppression factor and $\left(1 - 4r_\ell^2/z\right)^{3/2}$ stands for the lepton pair in a $L = 1$ wave. The two-loop function $\mathcal{J}$ describes the transitions $(\pi^+\pi^-)_{0^{++}} \to \ell^+\ell^-$. Care is compulsory in computing this function for varying z, because of compensating IR divergences among the $\pi\pi$ and $\pi\pi\gamma$ cuts. Various numerical tests were performed, in particular analyticity of $\mathcal{J}$ as a function of z.

We have included the standard $K_L \to \pi^0\gamma\gamma$ parametrization (see [7]) with the obvious intent of taking the ratio $R_{\gamma\gamma} = \Gamma_{\ell^+\ell^-}/\Gamma_{\gamma\gamma}$. The crucial point is that for a large range of parametrization of the vertex $K_L \to \pi^0 P^+ P^-$, $R_{\gamma\gamma}$ is stable even if both the $\ell^+\ell^-$ and $\gamma\gamma$ spectra vary much. For dynamical reasons, the $e^+e^-, \mu^+\mu^-$ and $\gamma\gamma$ modes react similarly to modulations in the distribution of momentum entering the scalar subprocess (i.e., to $a^P(z)$). Given this observation, we infer the branching ratio of $\ell^+\ell^-$ modes from the experimental result for $\gamma\gamma$. In doing so, some higher order chiral corrections are included in our result, in particular the $\mathcal{O}\left(p^6\right)$ chiral counterterms (with their VMD contents) needed to describe both the rate and spectrum for $K_L \to \pi^0\gamma\gamma$. The stability of $R_{\gamma\gamma}$ is the key dynamical feature permitting such an extrapolation, and thereby getting a reliable estimation for $\Gamma_{\mu^+\mu^-}$.

Numerically, taking $B\left(K_L \to \pi^0\gamma\gamma\right)^{\exp} = (1.42 \pm 0.13) \times 10^{-6}$ as the average of the KTeV[2] and NA48[3] measurements, we find $B\left(K_L \to \pi^0\mu^+\mu^-\right)_{CPC}^{0^{++}} = (5.2 \pm 1.6) \times 10^{-12}$, with a conservative error estimate of 30%.

Finally, the differential rate is in Fig.2b, and shows that an appropriate kinematical cut can reduce the relative contamination of the CPC contribution to below 10%.

3 Phenomenological Analysis

The final parametrizations are, in the Standard Model ($\kappa = 10^4 \,\mathrm{Im}\,\lambda_t = 1.36 \pm 0.12$)

$$\begin{aligned}
B\left(K_L \to \pi^0 \nu\bar{\nu}\right) &\approx \left(16\kappa^2\right) \times 10^{-12} && [8] \\
B\left(K_L \to \pi^0 e^+ e^-\right) &\approx \left(2.4\kappa^2 \pm 6.2\left|a_S\right|\kappa + 15.7\left|a_S\right|^2\right) \times 10^{-12} && [4] \\
B\left(K_L \to \pi^0 \mu^+ \mu^-\right) &\approx \left(1.0\kappa^2 \pm 1.6\left|a_S\right|\kappa + 3.7\left|a_S\right|^2 + 5.2\right) \times 10^{-12} && [5]
\end{aligned}$$

The κ^2 terms are DCPV, the ICPV parameter a_S is the counterterm dominating $K_S \to \pi^0\ell^+\ell^-$, recently measured by NA48/1 as $\left|a_S^{\exp}\right| = 1.2 \pm 0.2$[1]. The interference between DCPV and ICPV has been argued to lead to positive sign[4, 9].

It is interesting to keep track of the underlying short-distance physics, especially for analyzing the possible impact of New Physics. The FCNC diagrams in Fig.1a,b, including QCD corrections, leads to the effective Hamiltonian [10]

$$H_{eff}^{|\Delta S|=1} = \frac{G_F \alpha}{\sqrt{2}} V_{ts}^* V_{td} \left(y_{7V} Q_{7V} + y_{7A} Q_{7A}\right) \quad \text{with } Q_{7V(A)} = \left(\bar{s}d\right)_{V-A} \otimes \left(\bar{\ell}\ell\right)_{V(A)}$$

with $y_{7V} = 0.73 \pm 0.04$ (for $\mu \simeq 1$ GeV) and $y_{7A} = -0.68 \pm 0.03$. The various CPV coefficient dependences on these Wilson coefficients are (ICPV is long-distance)

$$\begin{array}{lll}
C_{ICPV}^{e} = 15.7 & C_{int.}^{e} = 8.91 y_{7V} & C_{DCPV}^{e} = 2.67\left(y_{7A}^2 + y_{7V}^2\right) \\
C_{ICPV}^{\mu} = 3.7 & C_{int.}^{\mu} = 2.12 y_{7V} & C_{DCPV}^{\mu} = 0.63\left(y_{7A}^2 + y_{7V}^2\right) + 0.85 y_{7A}^2
\end{array}$$

There is a simple phase-space suppression factor for all terms, but somewhat compensating this, the muon mode DCPV receives an additional helicity-suppressed axial-vector FCNC contribution. This gives different sensitivity of the two modes to the underlying short-distance physics.

Plotting the muon mode against the electron one in terms of $\mathrm{Im}\,\lambda_t$ (see Fig.3), we get a curve whose spreading is directly related to the relative strength of the vector and axial vector FCNC. This plane is especially suited to study New Physics impacts. Let us take as an example the enhanced electroweak penguin model (EEWP) of Buras *et al*[11], which affects the Wilson coefficients as $y_{7V}^{EEWP} \approx 0.9$ and $y_{7A}^{EEWP} \approx -3.2$. Taking all errors into account, we get the theoretical predictions for positive interference as in Fig.3, with the corresponding branching ratios

	S.M. $\left(\times 10^{-11}\right)$	EEWP $\left(\times 10^{-11}\right)$ [11]	Experiment [12]
$K_L \to \pi^0\nu\bar{\nu}$[8]	3.0 ± 0.6	31 ± 10	$< 5.9 \times 10^{-7}$
$K_L \to \pi^0 e^+ e^-$	$3.7^{+1.1}_{-0.9}$	9.0 ± 1.6	$< 2.8 \times 10^{-10}$
$K_L \to \pi^0 \mu^+ \mu^-$	1.5 ± 0.3	4.3 ± 0.7	$< 3.8 \times 10^{-10}$

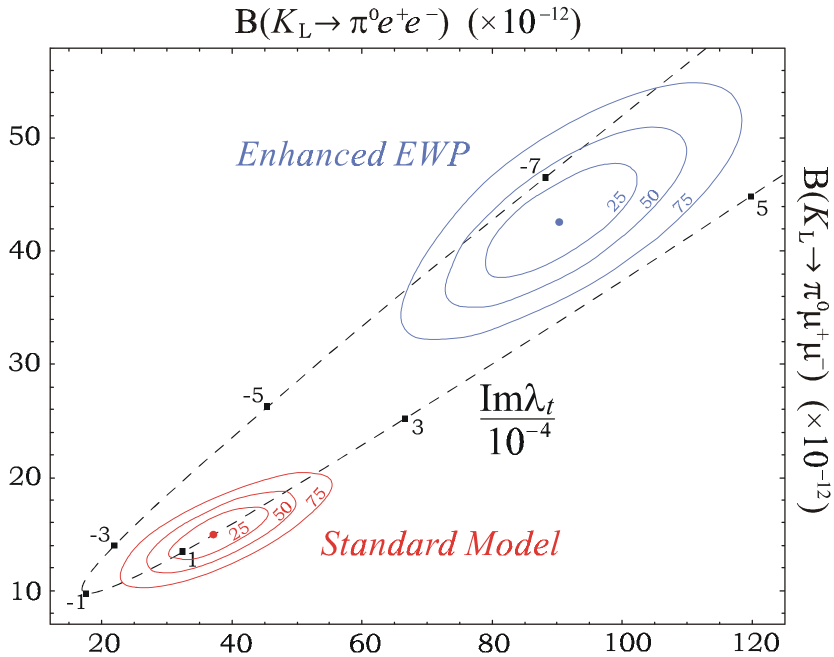

Figure 3: The confidence regions for the SM and EEWP scenario.

Note that the central value for EEWP is not on the $\operatorname{Im}\lambda_t$ curve as the ratio $(y_{7V}/y_{7A})^{SM} \neq (y_{7V}/y_{7A})^{EEWP}$.

In conclusion, the set of rare K_L decay modes $\pi^0\nu\bar{\nu}$, $\pi^0 e^+e^-$ and $\pi^0\mu^+\mu^-$ is now fully under theoretical control, and provides for a stringent test of the Standard Model. In addition, if a signal of New Physics is found, information on its nature can be extracted from the observed pattern of branching ratios.

Experimentally, it is clear that the $K_L \to \pi^0\mu^+\mu^-$ mode should be seriously considered. Also, additional measurements of $K_S \to \pi^0\ell^+\ell^-$ would be certainly very desirable since the main uncertainty on the theoretical prediction for the CPV parts, and thus the spreading of the confidence regions in Fig.3, comes from a_S.

4 Acknowledgements

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